Hola

Teorema de Noether

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$$\varepsilon \frac{\mathrm{d}}{\mathrm{d}t}\left[\sum_i \frac{\partial \mathcal{L}}{\partial \dot{q}_i}\eta_i - \left(\sum_i \frac{\partial \mathcal{L}}{\partial \dot{q}_i}\dot{q}_i - \mathcal{L}\right)\tau - F\right] = \Delta\mathcal{L} + \varepsilon\mathcal{L}\dot{\tau} - \varepsilon\frac{\mathrm{d}F}{\mathrm{d}t}$$

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Que lo podemos reescribir de forma más amigable:

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Si la transformación no involucra al tiempo ($\tau = 0$), queda la versión reducida del teorema:

$$\Delta\mathcal{L} = 0 \quad \Longrightarrow \quad I = \sum_i P_i\eta_i = \text{cte}$$